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An EOQ Model of Deteriorating Product with Variable Demand Rate under Trade Credit

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ABSTRACT:
In this paper, an EOQ model has been developed for deteriorating products with life time, linear inventory level dependent demand rate and time dependent demand rate under trade credit. Three components demand rate has been considered. Deterioration rate has been taken constant. Permissible delay in payments is allowed.

Key-words: Deterioration, demand rate, life time and trade credit.

INTRODUCTION:
In traditional inventory models the demand rate is assumed to be constant. It is observed that the stock level may influence the demand rate in the case of some consumer products. It is common experience that displaced stock level attracts more consumers. This observation inspired the researchers to consider the stock dependent demand rate. Several researchers such as **Level et al.** (1972), **Silver** (1981) and **Silver & Peterson** (1985) worked in this direction.

Gupta & Vrat (1986) discussed the stock dependent demand rate inventory model. Calculation of average system cost was not correct in this paper. **Mandal & Phaujdar** (1989) suggested the correction to the average system cost. **Bakar and Urban** (1988) provided the first rigorous attempt in developing the stock dependent demand rate. The functional form presented by them is realistic and logical from practical as well as economic point of view.

Dutta & Pal (1990) developed an inventory model with stock dependent demand rate using some functional form as taken by **Bakar and Urban** (1988). **Dutta & Pal** (1990) developed another model for deteriorating items with demand rate dependent on inventory level with shortages. **Mandal & Phaujdar** (1989) also developed another model for deteriorating items with stock dependent demand rate, variable rate of deterioration and shortages fully backlogged. **Sarkar, et al.** (1997) introduced the realistic concept of decreasing demand. **Jain and Kumar** (2007) developed an inventory model with stock level dependent demand rate, shortages and decrease in demand. They considered two level demand rate for infinite time horizon as well as finite time horizon. **Hari Kishan, Megha Rani and Deep Shikha** (2012) discussed the inventory model of deteriorating products with life time under declining demand and permissible delay in payment.

It is realistic situation that the demand rate of any product during life time, deteriorating period and shortage period are different. So in this paper, three components demand rate has been considered. The demand rate is different during three phases. The demand rate is decreasing in the three phases. Permissible delay in payments is allowed.

Notations and Assumptions:

Notations:

The following notations have been used in this paper:

$q(t)$: Inventory level at any time t.

S: Stock level at the beginning of each cycle after fulfilling the backorders. c_1: The holding cost per unit per unit time.
c_2: The shortage cost per unit per unit time.
C_H: The total holding cost of inventory in the interval $[0, T_0]$.
C_S: The total shortage cost of inventory in the interval $[T_0, T]$.
C_D: The total holding cost of inventory in the interval $[\mu, T_0]$.
μ: The time upto which there is no deterioration.
T_0: Time period in which there is inventory in the system.
T: The length of each cycle time.
c: Unit purchasing cost.
s: Unit selling price.
I_c: Interest charged per Re per unit time.
I_e: Interest earned per Re per unit time.

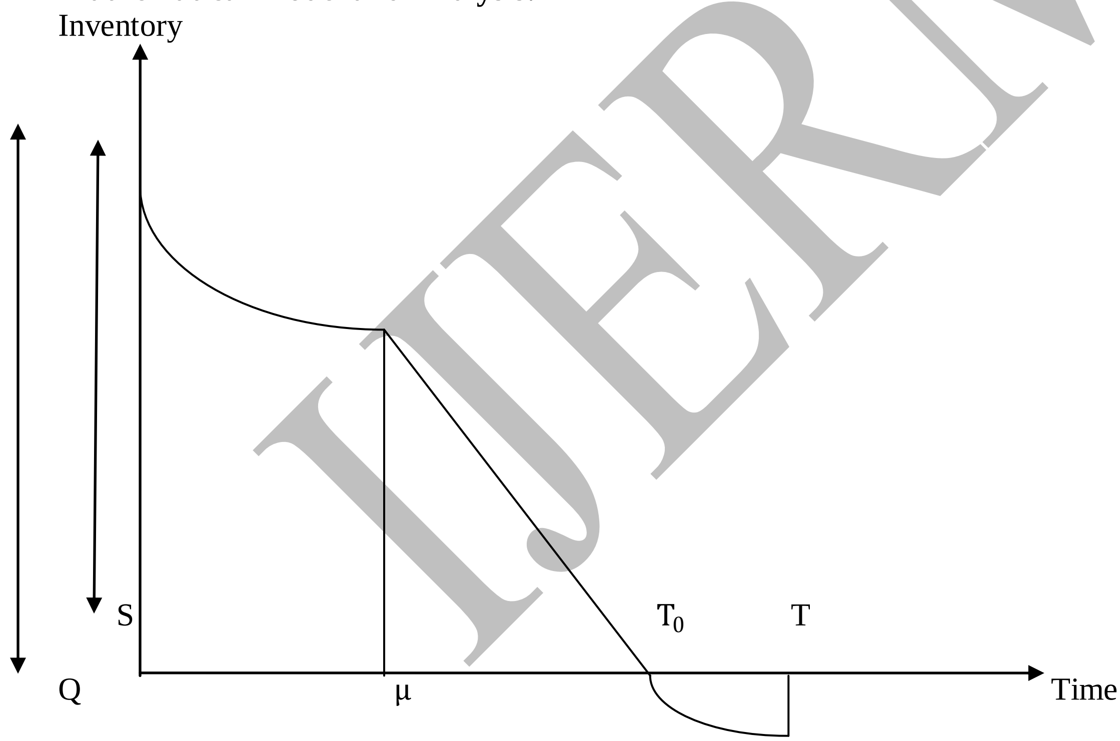
Assumptions:

1. There is single item in the inventory system.
2. Lead time is negligible. It is considered as zero.
3. Replenishments are instantaneous, i.e. the replenishment rate is taken infinite.
4. Shortages are allowed and fully backlogged.
5. Three components demand rate is deterministic and known function of instantaneous inventory level upto a certain interval of time and after that the demand rate is time dependent. Thus the demand rate is given by

$$\frac{dq}{dt} = \begin{cases} (\alpha + \beta q), & 0 \le t \le \mu \\ (\alpha + \gamma q), & \mu \le t \le T_0 \\ (\alpha + \delta(t - T_0)), & T_0 \le t \le T \end{cases}$$

where $\alpha, \delta > 0$ and $0 < \gamma < \beta < 1$.

Mathematical Model and Analysis:



(Figure 1)

Let Q be the number of items received at the beginning of the cycle. $(Q - S)$ items are delivered for the fulfillment of backorder leaving a balance of S as the initial inventory of new cycle. The inventory level depleted at a rate of αq^{β} during the period $[0, \mu]$. During the period $[\mu, T_0]$ the inventory level depleted at the rate α. The inventory level falls to zero at time T_0. Shortages are allowed for replenishment upto time T. The model has been shown in the (Figure 1) given above.

The inventory system is governed by the following differential equations:

$$\frac{dq}{dt} = -(\alpha + \beta q), \qquad 0 \le t \le \mu \qquad \ldots(1)$$

$$\frac{dq}{dt} + \theta q = -(\alpha + \gamma q), \qquad \mu \le t \le T_0 \qquad \ldots(2)$$

$$\frac{dq}{dt} = -(\alpha + \delta(t - T_0)), \qquad T_0 \le t \le T \qquad \ldots(3)$$

With the following boundary conditions:

$$q(0) = S,\ q(T_0) = 0 \text{ and } q(T) = -(Q - S). \qquad \ldots(4)$$

Solution of equations (1), (2) and (3) with the help of boundary conditions (4) are respectively given by

$$q = \left(S + \frac{\alpha}{\beta}\right) e^{-\beta t} - \frac{\alpha}{\beta}, \qquad 0 \le t \le \mu \qquad \ldots(5)$$

$$q = \frac{\alpha}{\theta+\gamma}\left[e^{(\theta+\gamma)(T_0 - t)} - 1\right], \qquad \mu \le t \le T_0 \qquad \ldots(6)$$

$$q = -\alpha(t - T_0) + \frac{\delta}{2}(t^2 - T_0^2). \qquad T_0 \le t \le T \qquad \ldots(7)$$

From expressions (5) and (6), we get

$$S = \frac{\alpha}{\theta+\gamma}\left[e^{(\theta+\gamma)(T_0 - \mu)} - 1\right] e^{\beta\mu} + \frac{\alpha}{\beta}(e^{\beta\mu} - 1). \qquad \ldots(8)$$

Annual ordering cost$=\frac{A}{T}$.

The total holding cost during the inventory cycle is given by

$$C_H = c_1\left[\int_0^{T_0} q\,dt\right]$$

$$= c_1\left[\int_0^{\mu} q\,dt + \int_{\mu}^{T_0} q\,dt\right]$$

$$= c_1\left[\left(S + \frac{\alpha}{\beta}\right)\frac{(1 - e^{-\beta\mu})}{\beta} - \frac{\alpha\mu}{\beta} + \frac{\alpha}{\theta+\gamma}\left\{\frac{(1 - e^{(\theta+\gamma)(T_0-\mu)})}{\theta+\gamma} - (T_0 - \mu)\right\}\right]. \qquad \ldots(9)$$

Total shortage cost for the entire cycle is

$$C_S = c_2\left[\int_{T_0}^{T} q\,dt\right]$$

$$= c_2\left[\int_{T_0}^{T}\left\{-\alpha(t - T_0) + \frac{\delta}{2}(t^2 - T_0^2)\right\} dt\right]$$

$$= c_2\left[\frac{\alpha T^3}{6} - \frac{\alpha T^2}{2} + \alpha T_0 T\left(1 - \frac{\delta T_0}{2}\right) - \frac{\alpha T_0^2}{2} + \frac{\alpha T_0^3}{3}\right]. \qquad \ldots(10)$$

Total deteriorated units per cycle are given by

$$D = S - \int_{\mu}^{T_0}(\alpha + \gamma q)\,dt$$

$$= S - \int_{\mu}^{T_0}\left(\alpha + \frac{\alpha\gamma}{\theta+\gamma}\left[e^{(\theta+\gamma)(T_0 - t)} - 1\right]\right) dt$$

$$= S - \frac{\alpha\theta}{(\theta+\gamma)}(T_0 - \mu) - \frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)} - 1\right). \qquad \ldots(11)$$

The total amount backordered at the end of each cycle is given by

$$Q - S = \alpha(T - T_0) - \frac{\delta}{2}(T^2 - T_0^2). \qquad \ldots(12)$$

Now we consider the following two cases:

Case 1: $M \le T_0$.

Sub case I: $0 < M \le \mu$.

In this case, the interest payable is given by

$$I_{p1}^1 = cI_c\left[\int_M^\mu q(t) + \int_\mu^{T_0} q(t)\right]dt$$
$$= cI_c\left[\int_M^\mu\left(\left(S+\frac{\alpha}{\beta}\right)e^{-\beta t} - \frac{\alpha}{\beta}\right)dt + \int_\mu^{T_0}\frac{\alpha}{\theta+\gamma}\left(e^{(\theta+\gamma)(T_0-t)} - 1\right)dt\right]$$
$$= cI_c\left[\frac{1}{\beta}\left(S+\frac{\alpha}{\beta}\right)\left(e^{-\beta M} - e^{-\beta\mu}\right) - \frac{\alpha}{\beta}(\mu - M)\right.$$
$$\left. + \frac{\alpha}{(\theta+\gamma)^2}\left[e^{(\theta+\gamma)(T_0-\mu)} - 1\right] - (T_0-\mu)\right]. \qquad \ldots(13)$$

In this case, the interest earned is given by

$$I_{e1}^1 = sI_e\left[\int_0^\mu(\alpha+\beta q)tdt + \int_\mu^{T_0}(\alpha+\gamma q)tdt\right]$$
$$= sI_e\left[\int_0^\mu\left(\alpha+\beta\left(S+\frac{\alpha}{\beta}\right)e^{-\beta t} - \frac{\alpha}{\beta}\right)tdt + \int_\mu^{T_0}\left(\alpha+\gamma\frac{\alpha}{\theta+\gamma}\left[e^{(\theta+\gamma)(T_0-t)} - 1\right]\right)tdt\right]$$
$$= sI_e\left[\left(S+\frac{\alpha}{\beta}\right)\frac{1}{\beta^2}\left\{1-(\mu\beta-1)e^{-\beta\mu}\right\} + \frac{\alpha\theta\,(T_0-\mu)}{(\theta+\gamma)}\right.$$
$$\left. + \frac{\alpha\gamma}{(\theta+\gamma)^2}\left(\mu e^{(\theta+\gamma)(T_0-\mu)} - T_0\right) + \frac{\alpha\gamma}{(\theta+\gamma)^3}\left(e^{(\theta+\gamma)(T_0-\mu)} - 1\right)\right]. \quad \ldots(14)$$

Therefore the total cost per unit time is given by

$$TC(1) = \frac{1}{T}\left[A + C_H + C_S + C_D + I_{p1}^1 - I_{e1}^1\right]$$
$$= \frac{A}{T} + \frac{c_1}{T}\left[\left(S+\frac{\alpha}{\beta}\right)\frac{(1-e^{-\beta\mu})}{\beta} - \frac{\alpha\mu}{\beta} + \frac{\alpha}{\theta+\gamma}\left\{\frac{(1-e^{(\theta+\gamma)(T_0-\mu)})}{\theta+\gamma} - (T_0-\mu)\right\}\right]$$
$$+ \frac{c_2}{T}\left[\frac{\alpha T^3}{6} - \frac{\alpha T^2}{2} + \alpha T_0 T\left(1-\frac{\delta T_0}{2}\right) - \frac{\alpha T_0^2}{2} + \frac{\alpha T_0^3}{3}\right]$$
$$+ \frac{c}{T}\left[S - \frac{\alpha\theta}{(\theta+\gamma)}(T_0-\mu) - \frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)} - 1\right)\right] + \frac{cI_c}{T}\left[\frac{1}{\beta}\left(S+\frac{\alpha}{\beta}\right)\left(e^{-\beta M} - e^{-\beta\mu}\right) - \frac{\alpha}{\beta}(\mu - M)\right.$$
$$\left. + \frac{\alpha}{(\theta+\gamma)^2}\left[e^{(\theta+\gamma)(T_0-\mu)} - 1\right] - (T_0-\mu)\right]$$
$$- \frac{sI_e}{T}\left[\left(S+\frac{\alpha}{\beta}\right)\frac{1}{\beta^2}\left\{1-(\mu\beta-1)e^{-\beta\mu}\right\} + \frac{\alpha\theta\,(T_0-\mu)}{(\theta+\gamma)}\right.$$
$$\left. + \frac{\alpha\gamma}{(\theta+\gamma)^2}\left(\mu e^{(\theta+\gamma)(T_0-\mu)} - T_0\right) + \frac{\alpha\gamma}{(\theta+\gamma)^3}\left(e^{(\theta+\gamma)(T_0-\mu)} - 1\right)\right].$$
$$\ldots(15)$$

Sub case II: $\mu < M \le T_0$.

In this case, the interest payable is given by

$$I_{p1}^2 = cI_c\left[\int_M^{T_0} q(t)\right]$$
$$= cI_c\left[\int_M^{T_0}\frac{\alpha}{\theta+\gamma}\left(e^{(\theta+\gamma)(T_0-t)} - 1\right)dt\right]$$
$$= cI_c\frac{\alpha}{(\theta+\gamma)^2}\left[e^{(\theta+\gamma)(T_0-M)} - 1\right] - (T_0-M)\Big]. \qquad \ldots(16)$$

In this case, the interest earned is given by

$$I_{e1}^2 = sI_e\left[\int_0^\mu(\alpha+\beta q)tdt + \int_\mu^{T_0}(\alpha+\gamma q)tdt\right]$$
$$= sI_e\left[\left(S+\frac{\alpha}{\beta}\right)\frac{1}{\beta^2}\left\{1-(\mu\beta-1)e^{-\beta\mu}\right\} + \frac{\alpha\theta\,(T_0-\mu)}{(\theta+\gamma)}\right.$$
$$\left. + \frac{\alpha\gamma}{(\theta+\gamma)^2}\left(\mu e^{(\theta+\gamma)(T_0-\mu)} - T_0\right) + \frac{\alpha\gamma}{(\theta+\gamma)^3}\left(e^{(\theta+\gamma)(T_0-\mu)} - 1\right)\right]. \quad \ldots(17)$$

The total cost per unit time is given by

$$TC(2)=\frac{1}{T}\left[A+C_H+C_S+C_D+I_{p1}^2-I_{e1}^2\right]$$
$$=\frac{A}{T}+\frac{c_1}{T}\left[\left(S+\frac{\alpha}{\beta}\right)\frac{\left(1-e^{-\beta t}\right)}{\beta}-\frac{\alpha\mu}{\beta}+\frac{\alpha}{\theta+\gamma}\left\{\frac{\left(1-e^{(\theta+\gamma)(T_0-\mu)}\right)}{\theta+\gamma}-(T_0-\mu)\right\}\right]$$
$$+\frac{c_2}{T}\left[\frac{\alpha T^3}{6}-\frac{\alpha T^2}{2}+\alpha T_0 T\left(1-\frac{\delta T_0}{2}\right)-\frac{\alpha T_0^2}{2}+\frac{\alpha T_0^3}{3}\right]$$
$$+\frac{c}{T}\left[S-\frac{\alpha\theta}{(\theta+\gamma)}(T_0-t)-\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-t)}-1\right)\right]$$
$$+\frac{cI_c}{T}\frac{\alpha}{(\theta+\gamma)^2}\left[e^{(\theta+\gamma)(T_0-M)}-1\right]-(T_0-M)\Big]$$
$$-\frac{sI_e}{T}\left[\left(S+\frac{\alpha}{\beta}\right)\frac{1}{\beta^2}\left\{1-(\mu\beta-1)e^{-\beta\mu}\right\}+\frac{\alpha\theta\,(T_0-\mu)}{(\theta+\gamma)}\right.$$
$$\left.+\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(\mu e^{(\theta+\gamma)(T_0-\mu)}-T_0\right)+\frac{\alpha\gamma}{(\theta+\gamma)^3}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right)\right].$$

…(18)

Case 2: $M\geq T_0$.

In this case, there will be no interest charged.

The interest earned is given by

$$I_{e2}^1=sI_e\left[\int_0^{\mu}(\alpha+\beta q)tdt+\int_{\mu}^{T_0}(\alpha+\gamma q)tdt+(M-T_0)\int_{T_0}^{M}(\alpha+\delta(t-T_0))tdt\right]$$
$$=sI_e\left[\left(S+\frac{\alpha}{\beta}\right)\frac{1}{\beta^2}\left\{1-(\mu\beta-1)e^{-\beta\mu}\right\}+\frac{\alpha\theta\,(T_0-\mu)}{(\theta+\gamma)}\right.$$
$$+\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(\mu e^{(\theta+\gamma)(T_0-\mu)}-T_0\right)+\frac{\alpha\gamma}{(\theta+\gamma)^3}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right)$$
$$\left.+(M-T_0)^2\left\{\alpha+\frac{\delta}{2}(M-T_0)\right\}\right].\qquad\text{…(19)}$$

The total cost per unit time is given by

$$TC(3)=\frac{1}{T}\left[A+C_H+C_S+C_D-I_{e2}^1\right]$$
$$=\frac{A}{T}+\frac{c_1}{T}\left[\left(S+\frac{\alpha}{\beta}\right)\frac{\left(1-e^{-\beta\mu}\right)}{\beta}-\frac{\alpha\mu}{\beta}+\frac{\alpha}{\theta+\gamma}\left\{\frac{\left(1-e^{(\theta+\gamma)(T_0-\mu)}\right)}{\theta+\gamma}-(T_0-\mu)\right\}\right]$$
$$+\frac{c_2}{T}\left[\frac{\alpha T^3}{6}-\frac{\alpha T^2}{2}+\alpha T_0 T\left(1-\frac{\delta T_0}{2}\right)-\frac{\alpha T_0^2}{2}+\frac{\alpha T_0^3}{3}\right]$$
$$+\frac{c}{T}\left[S-\frac{\alpha\theta}{(\theta+\gamma)}(T_0-\mu)-\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-t)}-1\right)\right]$$
$$-\frac{sI_e}{T}\left[\left(S+\frac{\alpha}{\beta}\right)\frac{1}{\beta^2}\left\{1-(\mu\beta-1)e^{-\beta\mu}\right\}+\frac{\alpha\theta\,(T_0-\mu)}{(\theta+\gamma)}\right.$$
$$+\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(\mu e^{(\theta+\gamma)(T_0-\mu)}-T_0\right)+\frac{\alpha\gamma}{(\theta+\gamma)^3}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right)$$
$$\left.+(M-T_0)^2\left\{\alpha+\frac{\delta}{2}(M-T_0)\right\}\right].\qquad\text{…(20)}$$

Thus we have

$$TC=\begin{cases}TC(1), & 0<M\leq\mu\\ TC(2), & \mu<M\leq T_0\\ TC(3), & M\geq T_0\end{cases}\qquad\text{…(21)}$$

For optimization, we have

$$\frac{\partial TC}{\partial T_0}=0, \frac{\partial TC}{\partial T}=0 \text{ and } \left(\frac{\partial^2 TC}{\partial {T_0}^2}\right).\left(\frac{\partial^2 TC}{\partial T^2}\right)-\left(\frac{\partial^2 TC}{\partial T_0 \partial T}\right)^2 \geq 0. \qquad \ldots(22)$$

Now $\frac{\partial TC(1)}{\partial T_0}=0$ implies that

$$\frac{c_1}{T}\left[-\frac{\alpha}{\theta+\gamma}\left\{\frac{e^{(\theta+\gamma)(T_0-\mu)}}{(\theta+\gamma)^2}+1\right\}\right]+\frac{c_2}{T}\left[\alpha T(1-\delta T_0)-\alpha T_0+\alpha T_0^2\right]$$
$$-\frac{c}{T}\left[\frac{\alpha\theta}{(\theta+\gamma)}+\frac{\alpha\gamma\, e^{(\theta+\gamma)(T_0-t)}}{(\theta+\gamma)}\right]+\frac{\alpha}{(\theta+\gamma)}\left[e^{(\theta+\gamma)(T_0-\mu)}\right]-1]$$
$$-\frac{sI_e}{T}\left[\frac{\alpha\theta}{(\theta+\gamma)}+\alpha\gamma\left(\mu e^{(\theta+\gamma)(T_0-\mu)}\right)+\frac{\alpha\gamma}{(\theta+\gamma)}\left(e^{(\theta+\gamma)(T_0-\mu)}\right)\right]=0.$$
$$\ldots(23)$$

and $\frac{\partial TC(1)}{\partial T}=0$ implies that

$$A+c_1\left[\left(S+\frac{\alpha}{\beta}\right)\frac{(1-e^{-\beta\mu})}{\beta}-\frac{\alpha\mu}{\beta}+\frac{\alpha}{\theta+\gamma}\left\{\frac{(1-e^{(\theta+\gamma)(T_0-\mu)})}{\theta+\gamma}-(T_0-\mu)\right\}\right]$$
$$+\left[-\frac{\alpha T^3}{3}+\frac{\alpha T^2}{2}+\frac{\alpha T_0^2}{2}-\frac{\alpha T_0^3}{3}\right]$$
$$+c\left[S-\frac{\alpha\theta}{(\theta+\gamma)}(T_0-\mu)-\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right)\right]+c_2I_C\left[\frac{1}{\beta}\left(S+\frac{\alpha}{\beta}\right)\left(e^{-\beta M}-e^{-\beta\mu}\right)-\frac{\alpha}{\beta}(\mu-M)\right.$$
$$\left.+\frac{\alpha}{(\theta+\gamma)^2}\left[e^{(\theta+\gamma)(T_0-\mu)}-1\right]-(T_0-\mu)\right]$$
$$-sI_e\left[\left(S+\frac{\alpha}{\beta}\right)\frac{1}{\beta^2}\left\{1-(\mu\beta-1)e^{-\beta\mu}\right\}+\frac{\alpha\theta\,(T_0-\mu)}{(\theta+\gamma)}\right.$$
$$\left.+\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(\mu e^{(\theta+\gamma)(T_0-\mu)}-T_0\right)+\frac{\alpha\gamma}{(\theta+\gamma)^3}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right)\right]=0.$$
$$\ldots(24)$$

Similarly $\frac{\partial TC(2)}{\partial T_0}=0$ implies that

$$\frac{c_1}{T}\left[-\frac{\alpha}{\theta+\gamma}\left\{e^{(\theta+\gamma)(T_0-\mu)}-1\right\}\right]+\frac{c_2}{T}\left[\alpha T(1-\delta T_0)-\alpha T_0+\alpha T_0^2\right]$$
$$+\frac{c}{T}\left[-\frac{\alpha\theta}{(\theta+\gamma)}-\frac{\alpha\gamma e^{(\theta+\gamma)(T_0-t)}}{(\theta+\gamma)}\right]+\frac{cI_c}{T}\frac{\alpha}{(\theta+\gamma)}\left[e^{(\theta+\gamma)(T_0-M)}\right]-1]$$
$$-\frac{sI_e}{T}\left[\frac{\alpha\theta}{(\theta+\gamma)}+\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(\mu(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)}-1\right)+\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)}\right)\right]=0,$$
$$\ldots(25)$$

and $\frac{\partial TC(2)}{\partial T}=0$ implies that

$$A+c_1\left[\left(S+\frac{\alpha}{\beta}\right)\frac{(1-e^{-\beta\mu})}{\beta}-\frac{\alpha\mu}{\beta}+\frac{\alpha}{\theta+\gamma}\left\{\frac{(1-e^{(\theta+\gamma)(T_0-\mu)})}{\theta+\gamma}-(T_0-\mu)\right\}\right]$$
$$+c_2\left[-\frac{\alpha T^3}{6}+\frac{\alpha T^2}{2}-\frac{\alpha T_0^2}{2}+\frac{\alpha T_0^3}{3}\right]$$
$$+c\left[S-\frac{\alpha\theta}{(\theta+\gamma)}(T_0-\mu)-\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right)\right]$$
$$+cI_c\frac{\alpha}{(\theta+\gamma)^2}\left[e^{(\theta+\gamma)(T_0-M)}-1\right]-(T_0-M)]$$
$$-sI_e\left[\left(S+\frac{\alpha}{\beta}\right)\frac{1}{\beta^2}\left\{1-(\mu\beta-1)e^{-\beta\mu}\right\}+\frac{\alpha\theta\,(T_0-\mu)}{(\theta+\gamma)}\right.$$
$$\left.+\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(\mu e^{(\theta+\gamma)(T_0-\mu)}-T_0\right)+\frac{\alpha\gamma}{(\theta+\gamma)^3}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right)\right]=0.$$
$$\ldots(26)$$

Similarly $\frac{\partial TC(3)}{\partial T_0} = 0$ implies that

$$\begin{aligned}
&c_1\left[-\frac{\alpha}{\theta+\gamma}\left\{e^{(\theta+\gamma)(T_0-\mu)}-1\right\}\right]+c_2\left[\alpha T(1-\delta T_0)-\alpha T_0+\alpha T_0^2\right]\\
&\quad +c\left[-\frac{\alpha\theta}{(\theta+\gamma)}-\frac{\alpha\gamma\, e^{(\theta+\gamma)(T_0-\mu)}}{(\theta+\gamma)}\right]-sI_e\left[\frac{\alpha\theta}{(\theta+\gamma)}\right.\\
&\quad +\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(\mu(\theta+\gamma)e^{(\theta+\gamma)(T_0-\mu)}-1\right)+\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)}\right)\\
&\quad \left.+(M-T_0)\left\{2\alpha+\frac{3\delta}{2}(M-T_0)\right\}\right]=0, \qquad \ldots(27)
\end{aligned}$$

and $\frac{\partial TC(3)}{\partial T} = 0$ implies that

$$\begin{aligned}
&A+c_1\left[\left(S+\frac{\alpha}{\beta}\right)\frac{(1-e^{-\beta\mu})}{\beta}-\frac{\alpha\mu}{\beta}+\frac{\alpha}{\theta+\gamma}\left\{\frac{(1-e^{(\theta+\gamma)(T_0-\mu)})}{\theta+\gamma}-(T_0-\mu)\right\}\right] \qquad -\left[\frac{\alpha T^3}{3}-\frac{\alpha T^2}{2}-\frac{\alpha T_0^2}{2}+\frac{\alpha T_0^3}{3}\right]\\
&\qquad\qquad +c\left[S-\frac{\alpha\theta}{(\theta+\gamma)}(T_0-\mu)-\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right)\right]\\
&\quad -sI_e\left[\left(S+\frac{\alpha}{\beta}\right)\frac{1}{\beta^2}\left\{1-(\mu\beta-1)e^{-\beta\mu}\right\}+\frac{\alpha\theta\,(T_0-\mu)}{(\theta+\gamma)}\right.\\
&\quad +\frac{\alpha\gamma}{(\theta+\gamma)^2}\left(\mu e^{(\theta+\gamma)(T_0-\mu)}-T_0\right)+\frac{\alpha\gamma}{(\theta+\gamma)^3}\left(e^{(\theta+\gamma)(T_0-\mu)}-1\right)\\
&\quad \left.+(M-T_0)^2\left\{\alpha+\frac{\delta}{2}(M-T_0)\right\}\right]=0. \qquad \ldots(28)
\end{aligned}$$

Similarly the values of second derivatives can be obtained. Solving equations (23), (25) and (27) we can obtain the values of $T_0^*(1)$, $T_0^*(2)$ and $T_0^*(3)$ respectively. Substituting these values in (24), (26) and (28) we can obtain the values of $T^*(1)$, $T^*(2)$ and $T^*(3)$ respectively. Now $T_0^*(1)$ and $T^*(1)$ are the optimal values of T_0 and T if $0 < M \le \mu$ and the second derivatives satisfy (22). $T_0^*(2)$ and $T^*(2)$ are the optimal values of T_0 and T if $\mu < M \le T_0$ and second derivatives satisfy (22). $T_0^*(3)$ and $T^*(3)$ are the optimal values of T_0 and T if $M \ge T_0$ and second derivatives satisfy (22). Thus we get the optimal values of T_0^* and T^*. Putting these values in equation (8) and (12) we can obtain the values of S^* and Q^*. Mathematika or matlab softwares can be used for illustration.

CONCLUDING REMARKS:

In this paper, an EOQ model has been developed for deteriorating product with life time and three component demand rate under permissible delay in payment. Cost minimization technique has been used to obtain the optimal values of different parameters.

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